Design constraints:

- Max angular velocity of bottom rotational stage
- Max angular acceleration of bottom rotational stage
- Max angular velocity of elevation upper stage
- Max angular acceleration of elevation upper stage
- Length of antenna we plan on using
- Mass of antenna we plan on using
- Mounting pattern of antenna
- Soft
    - Electronics housing/box easily accessible
    - Preferably circular 1' diameter base
    - Wait-high table to mount the base

Assume 0.17m (7 inches) wide.

<u>Selecting an elevator motor</u>

Inertia of a rod in the middle is 1/12 M L^2. We can treat the mass and antenna combo as a line in the middle. Thus our mass is 2*M_{rod}. Assume M_{rod} is 0.45 kg. Assume rod is 0.75 meters long. Thus moment of inertia of the antenna/counterbalance is

$$\frac{1}{12}(0.45 \cdot 2)(0.75 \cdot 2)^2$$

$$= 0.16875$$

Assume we want the elevation stage to rise from horizontal to vertical in ½ a second. Assume constant acceleration for this period. Thus it needs to traverse π/2 radians in 0.5 seconds. Thus

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}at^2$$

$$\theta_f = \frac{1}{2}at^2$$

$$\frac{2\theta_f}{t^2} = a$$

$$\frac{2\left(\frac{\pi}{2}\right)}{0.5^2}$$

$$= 12.5663706144$$

In other words, we must be capable of accelerating the motor at 12.566 radians s^{-2}. Torque is angular acceleration times moment of inertia, thus

$$\tau = \omega' I$$

$12.566(0.167)$

$= 2.098522$

So the motor must be able to produce 2.1 N m of torque, aka 210 N/cm. But because motor torque depends on motor velocity, let's calculate the top speed of the elevation stage.

$$v_f = v_0 + at$$

$$v_f = at$$

$12.566(0.5)$

$= 6.283$

6.3 rad s^{-1} is ~1 rps = 60 rpm.

So our final motor requirements are:

- In SI units: produce 2.1 N m of torque @ 6.3 rad/s
- In other units: produce 210 N cm of torque @ 60 rpm

Browsing stepper online, I went to hybrid stepper motor. Then to the Nema 23 faceplate size. I chose the
"E Series Nema 23 Bipolar 1.8deg 3.0Nm." Checking the datasheet we have about 225 N cm at 60 RPM which is perfect for direct drive. This motor is only $23, though it does weigh 1.8kg = 4lbs so it's definitely hefty.

We can go a 4:1 gear reduction with no penalty on torque. Therefore, we'd be very much within the operating conditions of the motor, even considering substantial torque reductions from heat.

I am going to hold off on calculating the rotational motor until I can find the moment of inertia of the upper stage (to calculate torque requirements). However, from an elegance standpoint, I think using the same motor would be pretty slick and it almost certainly has enough torque.

Designing a counterbalance

The base of the antenna is 4.1" away from the axis of rotation, so to be safe nothing can stick out further than 3" from axis of rotation. Assume we want our counterbalance to be a stick of steel parallel to axis of rotation, 3" away form axis of rotation. This means inertia is just MR^2. Thus we can solve for required mass of this rod

$$\frac{1}{3}(0.45)(0.75)^2 = (x)(0.076)$$

-> x = 1.12 kg

Assuming width of rod is 3" (0.0762 meters), and density of steel is 7700 kg/m^3, then the rod would need to be

$$1.12 = 0.0762 \cdot \pi x^2 \cdot 7770$$

-> x = 0.95 inches which of course we would just make 1”.

To summarize, if we have a 1” diameter, 3” long steel rod 3” parallel (but 3” off axis) to axis of rotation, this will counter balance the arm.